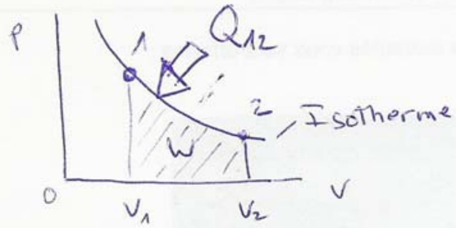
ad 4.3 Berechnung der Arbeit bei Zustandsänderungen

Isothermer Prozeß

$T = \text{konst}$ $\qquad p_1 \cdot V_1 = p_2 \cdot V_2 = m R T$



$$W = -\int_{V_1}^{V_2} p \, dV \qquad \text{aus } pV = mRT$$

$$= -\int_{V_1}^{V_2} \frac{mRT}{V} \, dV$$

$$= -mRT \int_{V_1}^{V_2} \frac{1}{V} \, dV \qquad \text{aus } \int \frac{1}{x} \, dx = \ln x$$

$$= -mRT \left[\ln V \right]_{V_1}^{V_2}$$

$$\underline{\underline{W = -mRT \left(\ln V_2 - \ln V_1 \right)}} \qquad \text{aus } \ln \frac{a}{b} = \ln a - \ln b$$

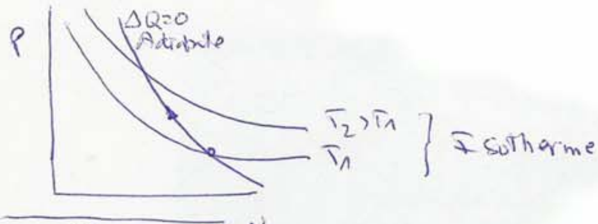
$$\underline{\underline{W = -mRT \ln \frac{V_2}{V_1}}} \qquad (1) \qquad \text{aus } p_1 V_1 = p_2 V_2$$

$$\frac{p_1}{p_2} = \frac{V_2}{V_1} \qquad \text{in (1)}$$

$$\underline{\underline{W = -mRT \ln \frac{p_1}{p_2}}}$$

Adiabaten Begriff $\Delta Q = 0$

$V\searrow \Rightarrow T\nearrow \Rightarrow p\nearrow$



$\boxed{p \cdot V^{\kappa} = konst}$ Adiabatengleichung

$1 < \kappa = \frac{c_p}{c_v} < \frac{5}{3}$

Arbeit

$$W = -\int_{V_1}^{V_2} p \cdot dV$$

$$= -\int_{V_1}^{V_1} \frac{konst}{V^{\kappa}} \cdot dV \qquad x^{-\kappa} = \frac{1}{x^{\kappa}}$$

$$= -konst \int_{V_1}^{V_2} V^{-\kappa} dV \qquad \text{aus } \int x^a dx = \frac{x^{a+1}}{a+1}$$

$$= -konst \left[\frac{V^{-\kappa+1}}{-\kappa+1} \right]_{V_1}^{V_2}$$

$$\underline{\underline{W = -\frac{konst}{1-\kappa} \left(V_2^{1-\kappa} - V_1^{1-\kappa} \right)}}$$

$$= -\frac{konst}{1-\kappa} V_1^{1-\kappa} \left(\frac{V_2^{1-\kappa}}{V_1^{1-\kappa}} - 1 \right)$$

$$W = -\frac{konst}{1-\kappa} V_1^{1-\kappa} \left[\left(\frac{V_1}{V_2} \right)^{\kappa-1} - 1 \right] \quad (1)$$

Aus

$$p_1 \cdot V_1^{\kappa} = Konst$$

$$p_1 \cdot V_1 \cdot V_1^{\kappa-1} = konst$$

$$V_1^{\kappa-1} = \frac{konst}{p_1 \cdot V_1}$$ mit idealem Gasgesetz

$$V_1^{\kappa-1} = \frac{konst}{mRT_1}$$

$$V_1^{1-\kappa} = \frac{mRT_1}{Konst}$$ in (1)

$$W = -\frac{konst}{1-\kappa} \cdot \frac{mRT_1}{konst} \left[\left(\frac{V_1}{V_2} \right)^{\kappa-1} - 1 \right]$$

$$\underline{\underline{W = -\frac{mRT_1}{1-\kappa} \left[\left(\frac{V_1}{V_2} \right)^{\kappa-1} - 1 \right]}} \quad (2)$$

Aus

$$p_1 V_1^{\kappa} = p_2 V_2^{\kappa}$$

$$\left(\frac{V_1}{V_2} \right)^{\kappa} = \frac{p_2}{p_1} \qquad / (\;)^{\frac{1}{\kappa}}$$

$$\frac{V_1}{V_2} = \left(\frac{p_2}{p_1} \right)^{\frac{1}{\kappa}}$$ in (2)

$$\underline{\underline{W = -\frac{mRT_1}{1-\kappa} \left[\left(\frac{p_2}{p_1} \right)^{\frac{\kappa-1}{\kappa}} - 1 \right]}}$$

Aus

$$\underline{p_1 V_1} \cdot V_1^{\kappa-1} = \underline{p_2 V_2} V_2^{\kappa-1}$$

$$mRT_1 \cdot V_1^{\kappa-1} = mRT_2 \; V_2^{\kappa-1}$$

$$\frac{V_1^{\kappa-1}}{V_2^{\kappa-1}} = \frac{T_2}{T_1}$$ in (2)

$$\underline{W = -\frac{mR}{1-\kappa} (T_2 - T_1)}$$